

### Experimental probability

It is possible to find the experimental probability,  $P(A)$ , of  $A$  occurring through an experiment.

The experiment should include a number of **trials** to see how often event  $A$  happens.

#### Key points

- $P(A)$  is the probability of event  $A$  happening.
- $P(A')$  is the probability of event  $A$  not happening.
- $P(A) = \frac{\text{number of times } A \text{ occurs}}{\text{total number of trials}}$

#### Example 1

Event  $A$  is that the same bird lands on Mrs Leung's bird table before 9 am each day. It does this on 40 days over a period of 1 year (365 days).

#### Reasoning

- Estimate the probability that tomorrow event  $A$  happens.
- Estimate the probability that tomorrow event  $A$  does not happen.

a  $P(A) = \frac{40}{365} = \frac{8}{73}$   
 b  $P(A') = \frac{325}{365} = \frac{65}{73}$

**Note:**  $P(A) + P(A') = 1$

#### Analysis

### Activity 1

A famous experiment in probability is Buffon's needle, in which a sewing needle is dropped onto a sheet of paper with parallel lines drawn on it that are spaced so that the distance between lines is the same as the length of the sewing needle. Event  $A$  is defined as the needle landing across a line.

- Take 15 needles (or 15 toothpicks). Measure the length of one of them. Now draw lines on a blank sheet of paper that are a distance apart that is the same length as your needle (or toothpick). Note: everyone in the class should have the same size needles or toothpicks.
- Drop the needles (or toothpicks) onto your sheet of paper and count how many cross a line.
- Record how many of the class's needles (or toothpicks) crossed a line and the total number of needles (or toothpicks) that were dropped by the entire class.
- Work out  $P(A)$ .

Complicated probability theory predicts that  $P(A) = \frac{2}{\pi} = 0.636\ 619\ 77\dots$

- Compare your class result with the one above. Was it close? You could record more drops to see if the result becomes closer to the expected probability of event  $A$ .



### Theoretical probability

If all possible outcomes are equally likely, you can work out how many of these outcomes should occur for event  $A$ . This is called calculating the theoretical probability of event  $A$ , which is written as  $P(A)$ .

#### Key points

- $P(A) = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

#### Example 2

A fair die is rolled. Calculate the probability of a prime number being rolled.

The prime numbers on a die are 2, 3 and 5.

Event  $A$  is the event of a roll giving a prime number.

$$P(A) = \frac{3}{6} \quad (\text{There are 3 desired outcomes, out of a total of 6 possible outcomes.})$$

$$= \frac{1}{2}$$

#### Example 3

A coin with heads on both sides is tossed. If event  $A$  is defined as the coin landing heads up, calculate  $P(A)$  and  $P(A')$ .

$$P(A) = \frac{2}{2} = 1 \quad (\text{a certainty})$$

$$P(A') = \frac{0}{2} = 0 \quad (\text{an impossibility})$$

#### Key points

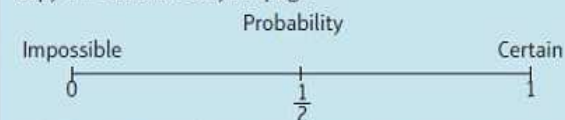
- If  $A$  is an event,  $0 \leq P(A) \leq 1$ .

#### Skills

##### Reasoning

### Activity 2

Copy this scale across your page.



Label the scale, marking approximately where you think the probability of the five events **A–E** should be placed.

- A hockey captain wins the toss of a coin at the start of a match ( $A$ ).
- A blue counter picked from a bag containing one blue, one red, one white and one green counters ( $B$ ).
- The blue counter is not picked from a bag containing one blue, one red, one white and one green counters ( $C$ ).
- You will be taken away by aliens on your way home from school today ( $D$ ).
- Your teacher will be wearing shoes for your next geography lesson ( $E$ ).

If  $A$  is an event, it either occurs ( $A$ ) or it does not occur ( $A'$ ).

It is certain that nothing else can happen.

## Exercise 5

Work out the following.



- 1 ►  $12 + 4 \times 2$     2 ►  $(12 + 4) \times 2$     3 ►  $11 - 3^2$   
 4 ►  $12 - 2^2 \times 3$     5 ►  $(8 - 3 \times 2)^2$     6 ►  $5 + (5 \times 2)^2 \div 5$   
 7 ►  $3 + 2 \div (7 - 9) \times (5 \times 2 - 6)$     8 ►  $\frac{4 + 4^2}{6 \div 3 \times 2}$   
 9 ► Insert brackets in this expression to make it correct:  $4 \times 5 - 3 + 2 = 10$   
 10 ► Insert brackets and symbols into this expression to make it correct:  $7 \ 5 \ 3 = 6$

## Exercise 5\*

Evaluate the following.



- 1 ►  $4 + 6 \times (2^2 + 5) \div 3 - 10$     2 ►  $2 - 5 \div (8 - 3) \times 2 + 8$   
 3 ►  $125 \div (7 \times 4 - 23)^2 \div 5$     4 ►  $\frac{3}{4} \div \frac{9}{10} \times \frac{4}{5} \div \frac{2}{3}$   
 5 ►  $1 + 10 \div 5 \times 11 - 3^2 \div 3$     6 ►  $(3 \times 4 \div 2^2 + 3) \times (6 \div 3 \times 5 - 5 \times 2 + 1) - 5$   
 7 ►  $\frac{1 + 4 \times 2}{6 - 1 \times 2} \div \frac{12 + 2^2}{8 \div 2 \times 2}$     8 ►  $\frac{\frac{2}{3} \times \frac{1}{4} + \frac{3}{4} \div \frac{9}{10}}{1 + 5 \times \frac{3}{5} - \frac{6}{7} \div \frac{3}{7}}$   
 9 ► Insert brackets in this expression to make it correct:  
 $8 - 2 + 1 \times 5 - 3 = 2$   
 10 ► Insert brackets and symbols in this expression to make it correct:  
 $8 \ 6 \ 2 \ 4 = 5$

## Significant figures and decimal places

If a piece of wood is to be cut 35.784 mm long, then this measurement is too accurate to mark out and cut, so 35.784 would be **rounded** to a suitable **degree of accuracy**. Numbers can be rounded to a certain number of significant figures or decimal places.

## Significant figures (s.f.)

The first s.f. is the first non-zero digit in a number, reading starting from the left to the right.

## Example 15

Highlight the first s.f. of the following numbers.

- a 27400    b 0.123    c 0.000583

## Skills

The first s.f. is highlighted in yellow.

## Interpretation

- a 27400    b 0.123    c 0.000583

When rounding to 2 s.f., look at the third s.f. If this is greater than or equal to 5, then round the second figure up. If rounding to 3 s.f., look at the fourth s.f. and so on.

## Example 16

Write

- a 1361    b 1350    c 1349    correct to 2 s.f.

## Skills

a 3rd s.f. is 6, and  $6 \geq 5$  so 3 rounds up to 4  $\Rightarrow 1361 = 1400$  (2 s.f.)  
 (1361 is closer to 1400 than 1300.)

## Interpretation

- b 3rd s.f. is 5, and  $5 \geq 5$  so 3 rounds up to 4  $\Rightarrow 1350 = 1400$  (2 s.f.)  
 (1350 is midway between 1400 and 1300, but we round up in this case.)  
 c 3rd s.f. is 4, and  $4 < 5$  so 3 is not rounded up  $\Rightarrow 1349 = 1300$  (2 s.f.)  
 (1349 is closer to 1300 than 1400.)

## Example 17

Write

- a 0.001361    b 0.00135    c 0.001349    correct to 2 s.f.

## Skills

a 3rd s.f. is 6, and  $6 \geq 5$  so 3 rounds up to 4  $\Rightarrow 0.001361 = 0.0014$  (2 s.f.)  
 (0.001361 is closer to 0.0014 than 0.0013.)

## Interpretation

- b 3rd s.f. is 5, and  $5 \geq 5$  so 3 rounds up to 4  $\Rightarrow 0.00135 = 0.0014$  (2 s.f.)  
 (0.00135 is midway between 0.0014 and 0.0013, but we round up in this case.)  
 c 3rd s.f. is 4, and  $4 < 5$  so 3 is not rounded up  $\Rightarrow 0.001349 = 0.0013$  (2 s.f.)  
 (0.001349 is closer to 0.0013 than 0.0014.)

## Decimal places (d.p.)

Count the digits the decimal point from left to right.  
 Rounding up or down follows the same rules as for s.f.

## Example 18

Write

- a 7.1361    b 0.135    c 0.0349    correct to 2 d.p.

## Skills

a 3rd d.p. is 6, and  $6 \geq 5$  so 3 rounds up to 4  $\Rightarrow 7.1361 = 7.14$  (2 d.p.)  
 (7.1361 is closer to 7.14 than 7.13.)

## Interpretation

- b 3rd d.p. is 5, and  $5 \geq 5$  so 3 rounds up to 4  $\Rightarrow 0.135 = 0.14$  (2 d.p.)  
 (0.135 is midway between 0.14 and 0.13, but we round up in this case.)  
 c 3rd d.p. is 4, and  $4 < 5$  so 3 is not rounded up  $\Rightarrow 0.0349 = 0.03$  (2 d.p.)  
 (0.0349 is closer to 0.03 than 0.04.)

## Skills

## Activity 3

## Interpretation

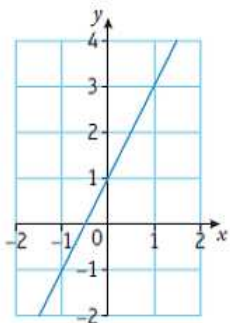
Use your calculator instruction book to find out how to:

- convert fractions to decimals and decimal to fractions
  - round to a certain number of significant figures or decimal places.
- Check by using the examples in this chapter.

**H** 3 ▶  $A$  is  $(1, 3)$  and  $B$  is  $(2, 6)$

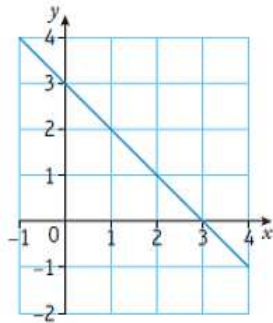
**H** 5 ▶  $A$  is  $(-2, 2)$  and  $B$  is  $(2, 1)$

6 ▶ Find the gradient of the graph.



**H** 4 ▶  $A$  is  $(-4, -1)$  and  $B$  is  $(4, 1)$

7 ▶ Find the gradient of the graph.



8 ▶ Draw a graph through the point  $(0, 0)$  with gradient 4.

9 ▶ A ladder reaches 12 m up a vertical wall and has a gradient of 4. How far is the bottom of the ladder from the wall?

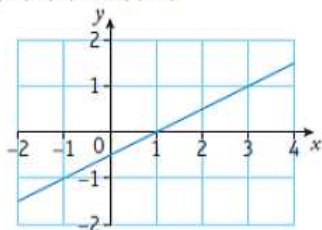
10 ▶ After take-off, an aeroplane climbs in a straight line with a gradient of  $\frac{1}{5}$ . When it has reached a height of 2000 m, how far has the aeroplane travelled horizontally?

Exercise 18\*

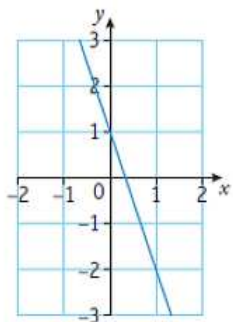
1 ▶ Find the gradient of the straight line joining  $A(-4, -1)$  to  $B(4, 2)$ .

2 ▶ Find the gradient of the straight line joining  $A(-3, 2)$  to  $B(4, -4)$ .

3 ▶ Find the gradient of the graph.  
**HINT** Choose two points to work with that have integer (whole number) coordinates.



4 ▶ Find the gradient of the graph.



5 ▶ The line joining  $A(1, 4)$  to  $B(5, p)$  has a gradient of 0.5. Find the value of  $p$ .

6 ▶ A line segment joins points  $A(-8, 0)$  and  $B(0, 0)$ . A line with a gradient of 0.75 is drawn from point  $A$  to point  $C$ . Complete the coordinates of point  $C(0, \underline{\quad})$  where the gradient crosses the  $y$ -axis.

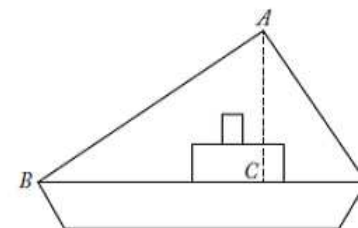
**HINT** Write the decimal gradient as a fraction.

7 ▶ A crane is lifting a boat suspended by ropes  $AB$  and  $AD$ .

The point  $C$  is vertically below  $A$ , and  $BC$  measures 5 m.

a The gradient of  $AB$  is 0.8. How high is  $A$  above  $C$ ?

b The gradient of  $AD$  is  $-1.25$ . What is the length of the boat?



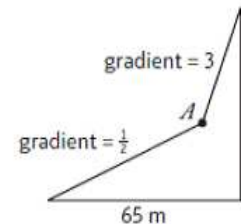
8 ▶ Do the points  $(1, 2)$ ,  $(51, 27)$  and  $(91, 48)$  lie on a straight line? Give reasons for your answer.

9 ▶ Find an algebraic expression for the gradient of the straight line joining  $A(p, q)$  to  $B(r, s)$ .

10 ▶ The line joining  $(3, p)$  to  $(7, -4p)$  is parallel to the line joining  $(-1, -3)$  to  $(3, 7)$ . Find  $p$ .

11 ▶ The gradient of the line joining  $(4, q)$  to  $(6, 5)$  is twice the gradient of the line joining  $(0, 0)$  to  $(4, q)$ . Find  $q$ .

12 ▶ One of the world's tallest roller coasters is in Blackpool, England. The maximum drop is 65 m over a horizontal distance of 65 m in two sections. The first section has a gradient of 3 and the second section has a gradient of  $\frac{1}{2}$ . How high is the point  $A$  above the ground?



**Skills**  
Reasoning

**Activity 1**

Find the gradient of  $AB$  as the point  $B$  moves closer and closer to the point  $C$ . Put your results in a table. What do you notice?



## Question 4

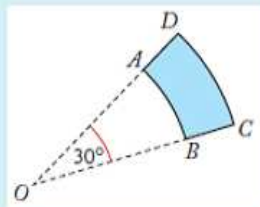
The diagram shows the sector of a circle  $ABCD$  with centre  $O$ .

Angle  $AOB = 30^\circ$

$OA$  is three times  $AD$ .

The perimeter of the shaded area is 18 cm.

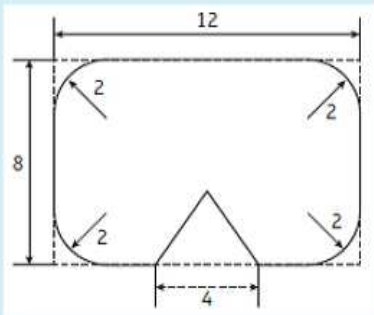
Find the shaded area in  $\text{cm}^2$ .



[4]

## Question 5

The paper label shown in the diagram is made from a rectangular sheet of paper measuring 8 cm by 12 cm. The corners are rounded by quarter-circle cuts of radius 2 cm. An equilateral triangle of side 4 cm is cut from the middle of one 12 cm side.



Calculate

- the percentage increase in perimeter from the original rectangle
- the percentage decrease in area from the original rectangle.

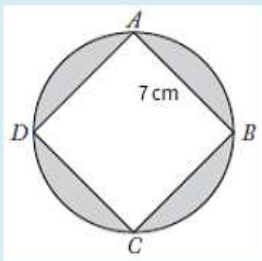
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[2]

## Question 6

$A, B, C$  and  $D$  are points on a circle.

$ABCD$  is a square of side 7 cm.



The area of the shaded region as a percentage of the whole circle area is  $m(\pi - 2)\%$ .

Show that  $m = \frac{100}{\pi}$

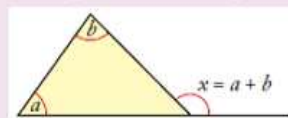
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[Total 25 marks]

# Chapter summary: Shape and Space 1

## Triangles

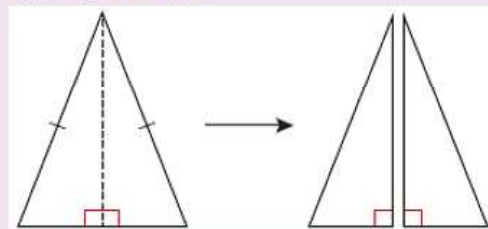
The angle sum of a triangle is  $180^\circ$ .



The exterior angle of a triangle equals the sum of the opposite angles.

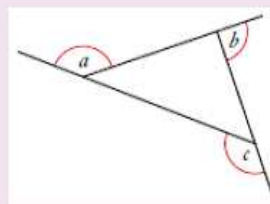
When doing problems, mark any angles you work out on a neat sketch of the diagram.

An isosceles triangle can be divided into two equal right-angled triangles.



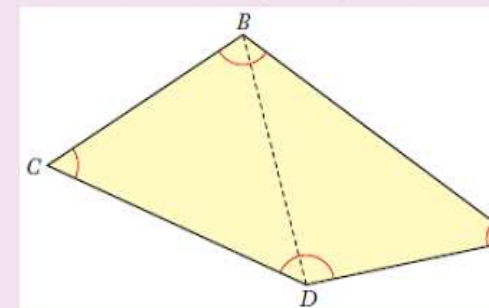
The sum of the exterior angles of a triangle is  $360^\circ$

$$a + b + c = 360^\circ$$

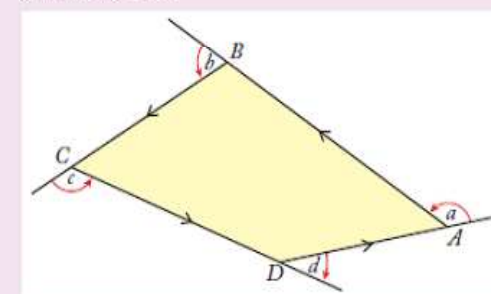


## Quadrilaterals

The sum of the interior angles of a quadrilateral is  $360^\circ$



The sum of the exterior angles of a quadrilateral is  $360^\circ$



## Pythagoras' theorem

$$a^2 = b^2 + c^2$$

Side  $a$  is always the hypotenuse.

The converse is also true.

If in a triangle  $a^2 = b^2 + c^2$  then the triangle is right-angled.

